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Carmeli's Accelerating Universe is Spatially Flat Without Dark Matter

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Carmeli's 5D brane cosmology has been applied to the expanding accelerating universe and it has been found that the distance redshift relation will fit the data of the high-z supernova teams without the need for dark matter. Also the vacuum energy contribution to gravity, Ω_Λ indicates that the universe is asymptotically expanding towards a spatially flat state, where the total mass/energy density $\Omega + \Omega_\Lambda \to 1$.

KEY WORDS: Carmeli's cosmology; dark matter; Hubble constant; redshift distance relation; critical density.

1. INTRODUCTION

The Carmeli cosmology (Carmeli, 2002) is as revolutionary in its implementation as it is in its interpretation. The metric used by Carmeli is unique in that it extends the number of dimensions of the universe by either one dimension if we consider only the radial velocity of the galaxies in the Hubble flow or by three if we consider all three velocity components. We will confine the discussion in this paper to only one extra dimension as does Carmeli. In that case the line element in five dimensions becomes

$$ds^2 = (1 + \Phi)c^2dt^2 - dr^2 + (1 + \Psi)\tau^2dv^2 \tag{1}$$

where $dr^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$ and Φ are Ψ potential functions to be determined. The time t is measured in the observers frame. The new dimension (v) is the radial velocity of the galaxies in the expanding universe, in accordance with Hubble flow. The parameter τ, the Hubble-Carmeli constant, is a constant at any epoch and its reciprocal (designated h) h is approximately the Hubble constant H_0.

The line element represents a spherically symmetric isotropic universe, and the expansion is the result of *space velocity* expansion. The expansion is observed

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at a definite time and thus $dt = 0$. Taking into account $d\theta = d\phi = 0$ (isotropy condition) and (1) becomes,

$$-dr^2 + (1+\Psi)\tau^2 dv^2 = 0 \tag{2}$$

This Eq. (given by Eq. B.38 and solved in Section B.10 in (Carmeli, 2002)) is reproduced here.

$$\frac{dr}{dv} = \tau\sqrt{1 + (1-\Omega)\frac{r^2}{c^2\tau^2}} \tag{3}$$

The parameter Ω is the mass/energy density of the universe expressed as a fraction of the critical or "closure" density. In this model $\rho_c = \frac{3}{8\pi G\tau^2} \sim 10^{-29}$ g cm^{-3}, ie $\Omega = \rho_m/\rho_c$ where ρ_m is the averaged matter/energy density of the universe.

Then (3) may be integrated exactly to get

$$r(v) = \frac{c\tau}{\sqrt{1-\Omega}}\sinh\left(\frac{v}{c}\sqrt{1-\Omega}\right)\forall\Omega \tag{4}$$

Carmeli has expanded (4) in the limit of small $z = v/c$ and small Ω to get

$$r(v) = \tau v\left(1 + (1-\Omega)\frac{v^2}{6c^2}\right) \tag{5}$$

$$\Rightarrow \frac{r}{c\tau} = z\left(1 + (1-\Omega)\frac{z^2}{6}\right) \quad \text{where} \quad \Omega < 1 \tag{6}$$

Thus we can write the expansion in terms of normalized or natural units (r/τ).

Equation (6) is plotted in Fig. 1 for various values of $\Omega = 1, 0.24$ and 0.03. Let us now re-write (4) in terms of natural units and for z small but arbitrary Ω

$$\frac{r(v)}{c\tau} = \frac{\sinh\left(z\sqrt{1-\Omega}\right)}{\sqrt{1-\Omega}}. \tag{7}$$

Equation (7) produces curves almost indistinguishable from (6) so this verifies that the approximations work for $z < 1$.

Now let us consider what happens to the density of matter as we look back in the cosmos with redshift, z. It was assumed in Fig. 1 that the value of Ω is fixed for each curve. Carmeli does this also in Figure A4, page 134 in ref (Carmeli, 2002). However, more correctly Ω varies as a function of z. For flat space we assume the following relation to hold,

$$\frac{\rho_m}{\rho_0} = (1+z)^3 = \frac{\Omega}{\Omega_0}, \tag{8}$$

where $\rho_m(z)$ is the averaged matter density of the universe at the redshift z, and ρ_0 is the averaged matter density of the universe locally $z \approx 0$. The parameter Ω_0 is

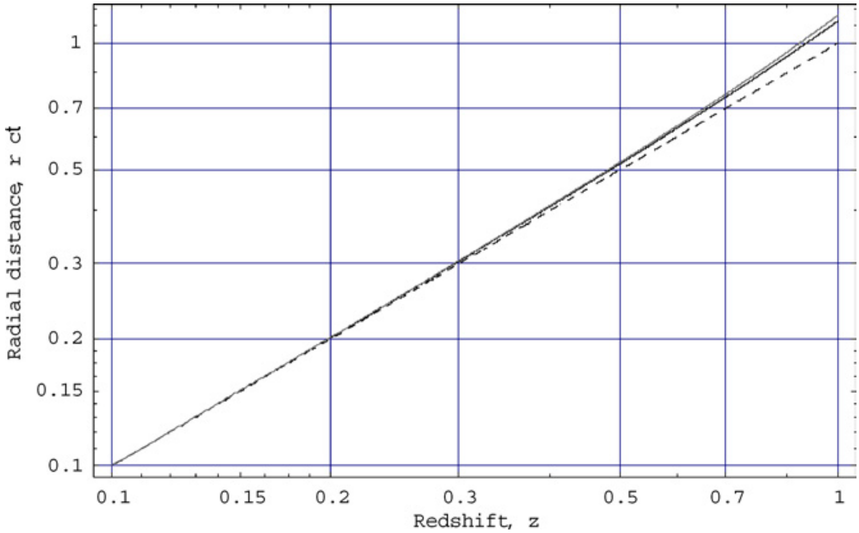

Fig. 1. Plot of (6), $r/c\tau$ vs redshift (z) for $\Omega = 1$ (broken line), 0.245 (solid black line) and 0.03 (solid grey line).

then the local averaged matter density expressed as a fraction of "closure" density. Eq. (8) results from the fact that as the redshift increases the volume decreases as $(1 + z)^3$. Notice at $z = 1$ that the universe is 8 times smaller in volume and therefore 8 times more dense, that is, at $z = 1$, $\Omega = 8$. Ω_0
Substituting (8) into (7) we get

$$\frac{r(v)}{c\tau} = \frac{\sinh\left(z\sqrt{1 - \Omega_0(1+z)^3}\right)}{\sqrt{1 - \Omega_0(1+z)^3}}. \tag{9}$$

Carmeli was able simulate the form of the $0.1 < z < 1$ redshift data of (Riess *et al.*, 1998) published in 1998 which announced an accelerating universe following the observations of (Garnavich *et al.*, 1997; Perlmutter *et al.*, 1997). See Figure A4, page 134 in (Carmeli, 2002). But in fact he had predicted this in 1996 (Carmeli, 1996). So this means that Carmeli assumed a value of total matter (normal + dark matter) density $\Omega = 0.245$, which was the accepted value in 1998.

Now let's plot (7) with $\Omega = 0.245$ and (9) with $\Omega_0 = 0.03$. See Fig. 2. This means that my modified Eq. (9) with $\Omega_0 = 0.03$ gives the same result as Carmeli's unapproximated Eq. (7) with his assumed value of $\Omega = 0.245$, but this includes dark matter. In fact, comparing (7) and (9), a local matter density of only $\Omega_0 = 0.03$–0.04 is necessary to have agreement. This effectively eliminates the need for the existence of dark matter on the cosmic scale at least.

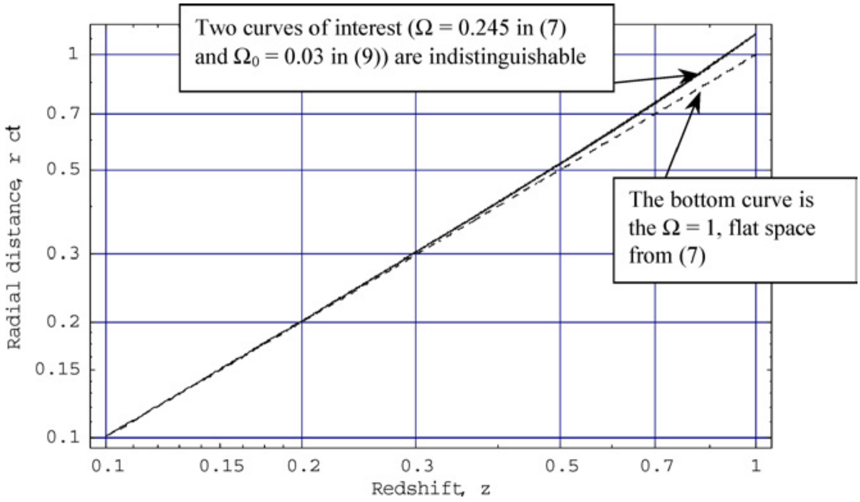

Fig. 2. Plot of (7) with $\Omega = 1$ (broken line) and 0.245 (solid black line) and (9) with $\Omega_0 = 0.03$ (solid grey line). Note: the top two curves lay on top of each other.

Table I shows the critical data from the comparison at redshifts between $z = 0.25$ and $z = 1$. It can be seen that the difference between the two equations over the domain of the measurements is much less significant than the fit to the data. If we assume $\Omega_0 = 0.04$ instead of $\Omega_0 = 0.03$, since both are within measured parameters, we get closer agreement at smaller redshifts but worse near $z = 1$.

In any case (7) and (9) really must be modified as $z \to 1$ to allow for relativistic effects, by replacing v/c with the relativistic form $\frac{v}{c} = \frac{(1+z)^2-1}{(1+z)^2+1}$. Therefore we can re-write (7) and (9) respectively as

$$\frac{r}{c\tau} = \frac{1}{\sqrt{1-\Omega}}\sinh\left(\frac{(1+z)^2-1}{(1+z)^2+1}\sqrt{1-\Omega}\right) \tag{10}$$

Table I. Comparison of Equations (7) and (9)

Redshift z	0.25	0.5	0.75	1.0
r/cτ from (7) with $\Omega = 0.245$	0.251984	0.515984	0.804591	1.13157
r/cτ from (9) with $\Omega_0 = 0.03$	0.252459	0.518935	0.810416	1.13157
% difference with $\Omega_0 = 0.03$	0.19	0.57	0.72	0.00
% difference with $\Omega_0 = 0.04$	0.17	0.43	0.23	1.28

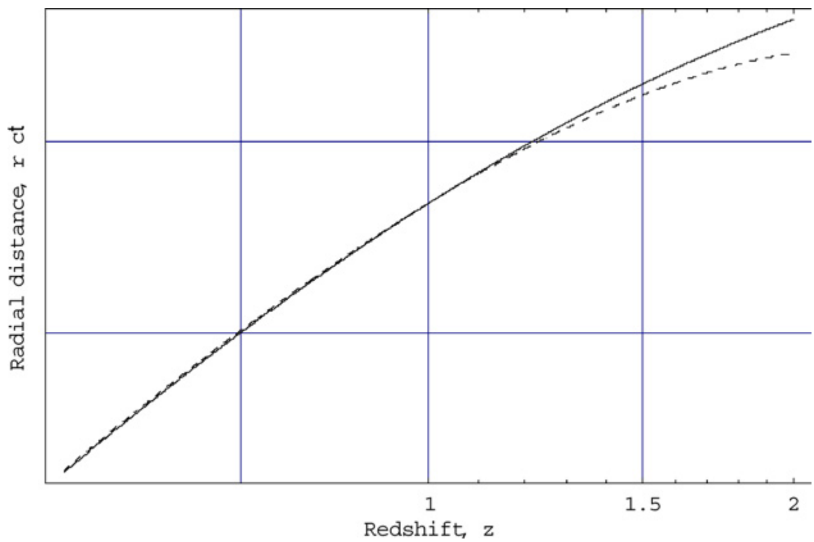

Fig. 3. Plot of (10) with $\Omega = 0.245$ (solid curve) and (11) with $\Omega_0 = 0.03$ (broken curve). Note: the two curves separate for $z > 1.2$.

and

$$\frac{r}{c\tau} = \frac{1}{\sqrt{1 - \Omega_0(1+z)^3}} \sinh\left(\frac{(1+z)^2 - 1}{(1+z)^2 + 1}\sqrt{1 - \Omega_0(1+z)^3}\right) \tag{11}$$

where the varying matter density has been taken into account. In Fig. 3, (10) and (11) are compared. The density approximation may be no longer valid past $z = 1$, because it is shown below that the vacuum energy term dominates and the universe is far from flat.

Based on the above analysis we can rewrite Eq. A.54 from (Carmeli, 2002) for H_0 as

$$H_0 = h\left[1 - (1 - \Omega_0(1+z)^3 \frac{z^2}{6}\right] \tag{12}$$

which according to Eq. A.51 from (Carmeli, 2002) may be further generalized without approximation, and using the relativistic form of the redshift. Still this eq. may only be approximate for $z > 1$ because of the density assumptions. However it becomes

$$H_0 = h\frac{\xi}{\sinh \xi} \quad \text{where} \quad \xi = \frac{(1+z)^2 - 1}{(1+z)^2 + 1}\sqrt{1 - \Omega_0(1+z)^3} \tag{13}$$

Both (12) and (13) have been plotted in Fig. 4, and for Carmeli's chosen value of $H_0 \approx 70\,\mathrm{kms}^{-1}\,\mathrm{Mpc}^{-1}$ at $z = 1$ in (12) yields $h \approx 80.2\,\mathrm{kms}^{-1}\,\mathrm{Mpc}^{-1}$ (very close

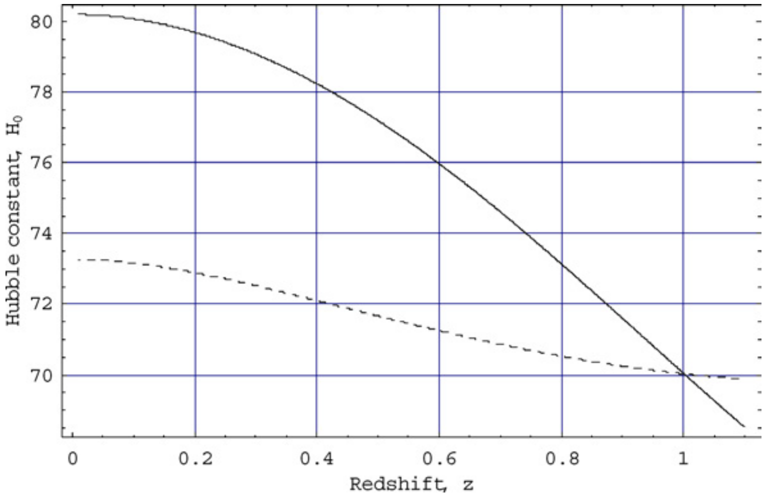

Fig. 4. Plot of (12) (solid curve) and (13) (broken curve). Note: the two curves intersect at $z = 1$ where $H_0 = 70$.

to Carmeli's value) but (13) yields $h \approx 73.27\,\mathrm{kms}^{-1}\,\mathrm{Mpc}^{-1}$. This means without the small z approximation the value of h is reduced when compared to that in (Carmeli, 2002).

The vacuum energy parameter Ω_Λ does not appear explicitly in Carmeli's model. It is only by a comparison with Friedmann-Lemaitre models can an assignment be made. On page 138 of (Carmeli, 2002) by comparing with the standard model it is shown that $\Omega_\Lambda = (H_0/h)^2$, therefore we can write

$$\Omega_\Lambda = \left(\frac{\sinh\xi}{\xi}\right)^{-2} \tag{14}$$

From (14) it is expected that using the unapproximated Eq. (13) for H_0 the value of Ω_Λ will be larger than Carmeli's value using the form of (6). Figure 5 shows the values for the vacuum energy density Ω_Λ (broken curve) and for the total energy density $\Omega_\Lambda + \Omega$ (solid curve) as a function of redshift, z. From (14) it follows that as the universe expands the total density tends to the vacuum energy density $\Omega_\Lambda \to 1$ (since $\Omega_0 \to 0$). This means a totally flat universe in a totally relaxed state. For small z the total density becomes

$$\Omega_\Lambda + \Omega \approx (1 + \Omega_0) + 3z\Omega_0 \tag{15}$$

It follows from (15) that for $\Omega_0 = 0.03$ at $z = 0$ the total density $\Omega_\Lambda + \Omega \approx 1.03$. This value is consistent with Carmeli's result of 1.009. However, it follows from (8) and (15) that the universe will always be open, $\Omega < 1$ as it expands. From Fig. 5 the total density $\Omega_\Lambda + \Omega$ is always greater than unity and as the universe

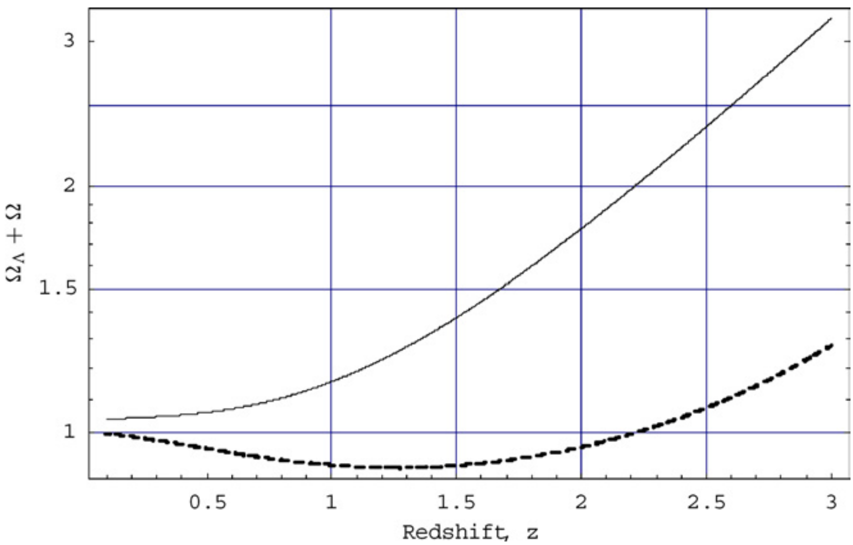

Fig. 5. Plot of Ω_Λ (broken curve) and total density $\Omega_\Lambda + \Omega$ (solid curve) as a function of redshift z. Notice that Ω_Λ tends to unity as z tends to zero and the total density tends to the local matter density Ω_0 + vacuum energy density Ω_Λ.

expands, it asymptotically approaches unity—therefore a spatially flat universe devoid of dark matter.

2. CONCLUSION

The 5D brane world of Moshe Carmeli has been has been applied to the expanding accelerating universe and the redshift distance relation has been generalised for redshifts up to at least $z = 1.2$. It has been found that if a certain form of the dependence of baryonic matter density on redshift is assumed then the resulting distance–redshift relation will fit the data of the high-z supernova teams without the need for dark matter. It is also seen that the vacuum energy contribution to gravity, Ω_Λ tends to unity as a function of decreasing redshift. Also since the baryonic matter density $\Omega_0 \to 0$ as the universe expands, the total mass/energy density $\Omega + \Omega_\Lambda \to 1$. This indicates that the universe, though always open because $\Omega < 1$, is asymptotically expanding towards a spatially flat state.

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